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Evidence of Stochastic Resonance in a Laser with Saturable Absorber: Experiment and Theory

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A laser with intracavity saturable absorber showing optical bistability is investigated through the simultaneous injection of modulation and noise on the pumping parameter; stochastic resonance is exhibited in the measured signal-to-noise ratio of the laser intensity.

KEY WORDS: Laser with a saturable absorber; stochastic resonance.

1. INTRODUCTION

A nonlinear phenomenon now commonly called stochastic resonance (SR) occurs when a small periodic signal superimposed on a broadband noise is processed by any sort of bistable system. If one measures the signal-to-noise ratio (SNR) of the output at the modulation frequency as a function of the injected noise, a curve with a peak appears. There is therefore a best noise level for which the bistable system acts as a selective amplifier in some range of frequencies.

This phenomenon was pointed out for the first time in 1981[(1)] in connection with the earth's ice ages. More recently there has been much theoretical work[(2,3)] particularly regarding the onset of SR with respect to different types of noise sources and potentials. The main interest lies first of all in the possibility to have, under particular conditions, an enhanced SNR for noisy signals processed by bistable filters with respect to the standard linear ones, and then in understanding the role that the noise can have in the transmission of information in physical as well as in biological systems.

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On another front, experimental evidence of the phenomenon of SR is not very common: an experiment using a Schmidt trigger as a bistable system[4] and analog simulations[5] are examples of observations in electronic devices. Applications to real physical systems include a modulated bidirectional ring laser,[6] an electron paramagnetic resonance system,[7] a free-standing magnetoelastic ribbon,[8] and passive optical systems.[9]

It has been claimed[6] that any system having a bistability or an hysteresis cycle should in principle show SR in the output signal. This is exactly the case of our laser, in which a bistability is present and can easily be observed as a function of either the pumping parameter or the laser frequency.

In this work we have investigated, for the first time, the occurrence of SR in a laser with intracavity saturable absorber (LSA) by adding a modulation plus noise to the pumping parameter. Furthermore, we give an account of the results by exploring theoretically a model of the LSA that well reproduces its behavior over a wide range of the control parameters.[10]

An LSA is a quantum optical device consisting of a laser cavity where an amplifying as well as an absorbing medium are placed.

The interaction between laser radiation and the two media strongly changes the behavior of the normal CO_2 laser, which admits, under proper conditions and as a function of the pumping parameter, only the ON and OFF operating modes, with laser intensity respectively equal to zero and different from zero. In the LSA, for different values of the control parameters (examples are absorber pressure, laser frequency, and pumping intensity), different time evolutions are observed for the output radiation intensity: constant, periodic, and aperiodic or chaotic.

The LSA is therefore a simple and reasonably well-controlled optical system showing nonlinear behavior. In the past years, a thorough experimental investigation of the aperiodic regimes has been performed and an interpretation in terms of homoclinic chaos has been given.[11]

In the LSA, optical bistability (OB) has also been observed,[12] that is, for an observable range of the mentioned control parameters, one can have the laser operating on two different stable modes, depending on the preceding time evolution.

The model we used for the theory reproduces both these behaviors with a very good qualitative agreement and with a partially quantitative one.

The work is organized in the following way: in Section 2 we describe the experiment, in Section 3 we report the experimental results, and in Section 4 we present a theoretical analysis.

2. DESCRIPTION OF THE EXPERIMENT

A detailed description of the experimental setup shown in Fig. 1 can be found in ref. 11 and in references therein; here we will give only a brief summary.

In our laser, the amplifying medium is CO_2, which has several lasing lines in the infrared region between 9 and 11 μm, and the gaseous saturable absorber is SF_6; the population inversion in the amplifier is obtained by an electric discharge, so the current intensity in this discharge plays the role of the pumping parameter. Through a reflection grating posed at one side of the optical cavity and an iris in the middle, the LSA can operate on single line and single mode (TEM_{00}). The laser radiation is monitored through a fast HgCdTe detector, with risetime <50 nsec. The electric discharge in the amplifying cell is driven by an HV current amplifier with a bandwidth larger than 50 kHz. The noise generator, which has a bandwidth of 100 kHz, and the modulation are summed and sent to the current amplifier.

The output of the detector and the discharge current are monitored simultaneously through a digital oscilloscope connected to a PC on which records of 32,000 points with 8-bit precision can be transferred simultaneously from both channels. An anti-aliasing filter was introduced when necessary for a successive calculation of the fast Fourier transform (FFT) from the recorded signal. The typical sampling frequency we used for our data was 5 kHz.

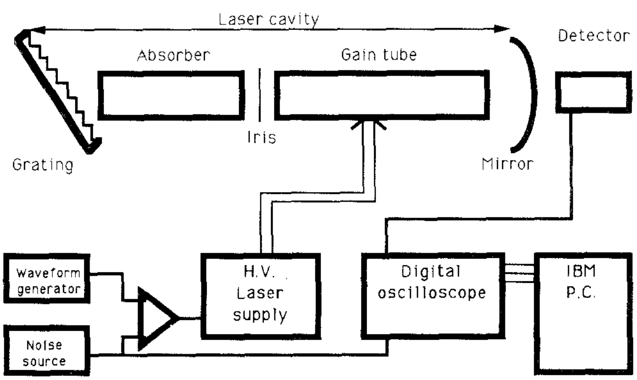

Fig. 1. Experimental apparatus. The laser cavity, defined by a reflection grating and a semi-reflecting mirror, contains two cells for the absorber and the amplifier. The modulation and the quasi-Gaussian noise are summed and sent to the electric discharge of the amplifying medium through a current amplifier; the radiation is monitored by an HgCdTe detector connected to a digital oscilloscope. All the acquisition is controlled by a PC.

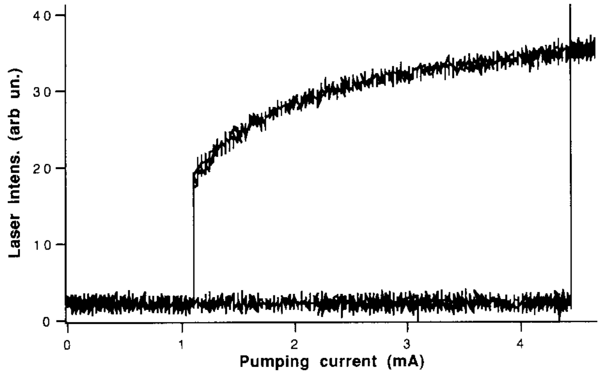

Fig. 2. Recording of the optical bistability relative to the LSA on the 10P(16) line with 20 mTorr SF_6; the current modulation is a sinusoidal wave at 0.6 Hz frequency.

The noise generator[13] is a dichotomic noise generator with clock frequency of about 5 MHz and a repetition period of some days. Through integration, the noise becomes Gaussianly distributed with a finite bandwidth. The frequency cutoff is larger than that of the current amplifier and both are much larger than the characteristic frequencies of our system, so the noise can be reasonably considered to be white.

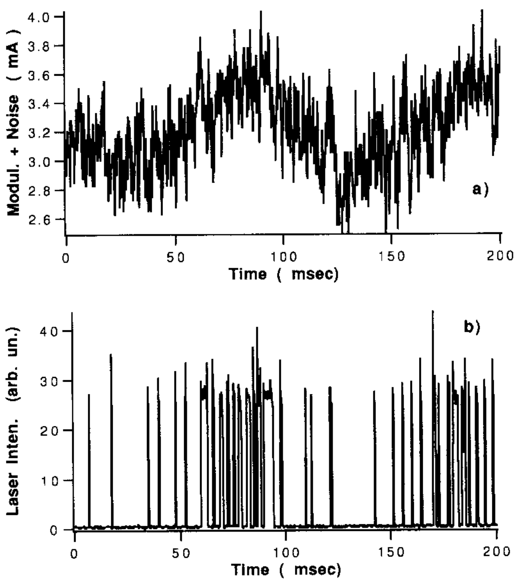

Fig. 3. (a) Noisy modulation at 10 Hz frequency and (b) bistable signal of the LSA operating on the CO_2 10P(16) line with 20 mTorr SF_6.

OB in our laser can be observed by varying different control parameters: absorber pressure, laser frequency (i.e., cavity length), and pumping parameter (i.e., current intensity). We chose to act on the pumping because it is the easiest to control. An example of OB between the ON and OFF solutions as a function of the pumping parameter is reported in Fig. 2.

Our proposal was to investigate the occurrence of stochastic resonance on the switching signal by simultaneously applying a modulation and a Gaussian white noise on the pump parameter (current intensity of the electric discharge).

The laser was set to operate inside the bistability region; then we added a modulation amplitude small enough not to cause a periodic switching between ON and OFF states; as the noise on the pumping parameter is increased, random switches occur at increasing rate.

The output laser intensity and the injected noise are then recorded for about 150 periods of the modulation and stored in the PC. A typical noisy modulation with the corresponding bistable signal is shown in Fig. 3.

3. RESULTS

From the recorded bistable signal we calculated the FFT and measured the signal-to-noise ratio (SNR) defined as the amplitude of the FFT component at the modulation frequency minus the background noise and divided by it. The background noise at the modulation frequency was evaluated from the neighbouring bins. In the case the signal was spread over multiple bins of the spectrum, we summed over the bins. From the recorded noisy signal we measured the noise variance. We reported the signal and the SNR as functions of the pump noise N_P defined as the variance of the noisy modulation taken at the output of the current amplifier. In all our data the SNR shows the typical behavior of stochastic resonance, as reported in Fig. 4.

We repeated our measurements for different values of the modulation frequency; the phenomenon seems to be robust in the range 0–100 Hz and the SNR peak has a dependence on the modulation frequency similar to that predicted by SR theory, as shown in Fig. 5. In fact, for increasing modulation frequencies, the peak occurs at bigger noise intensities and decreases in value. Peaks in the FFT at integer multiples of the modulation frequency are also observed for spectra in the SR zone.

Here we must comment that our spectra are not averaged spectra, because our system has a slight drift in time due to the thermic variations that modify the width of the bistability zone. The time scale of this variation is about 10 min, so we cannot extend a series of measures over this time interval.

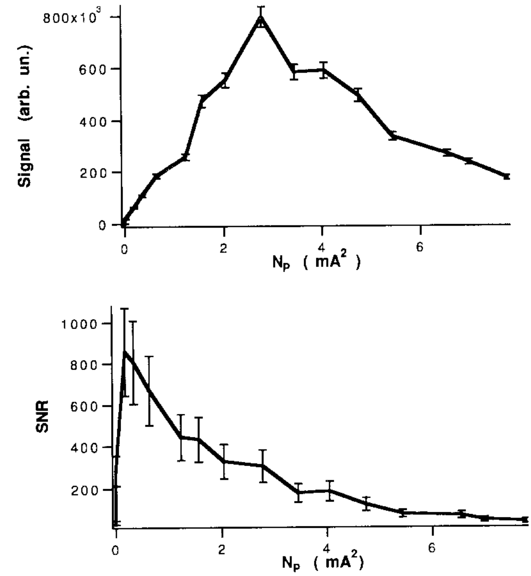

Fig. 4. Signal and signal-to-noise ratio response of the system for increasing values of the ratio of the noise to the amplitude modulation. Line 10P(16) with 20 mTorr SF_6; modulation frequency, 38.5 Hz; noise cut at 100 kHz.

Another control we made in our data was to process the records, via software, in a two-state filter so that any "intrawell motion" was eliminated. The computed SNR of the filtered signal did not differ from the original one and showed the same resonance peak.

As last quantities, we measured the residence time in each state and the first return time defined as the sum of two subsequent residence times in the ON and in the OFF state. It turns out (see Fig. 6) that in the SR zone, defined by Fig. 4, the first return times are distributed mainly at integer multiples of the modulation frequency.

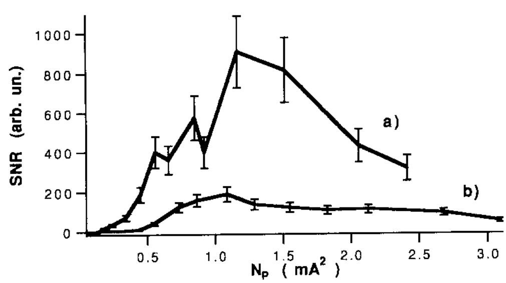

Fig. 5. Signal-to-noise ratio for different modulation frequencies: (a) 10 Hz, (b) 100 Hz. The laser conditions are the same as those of Fig. 4; the sampling frequency is 5 kHz.

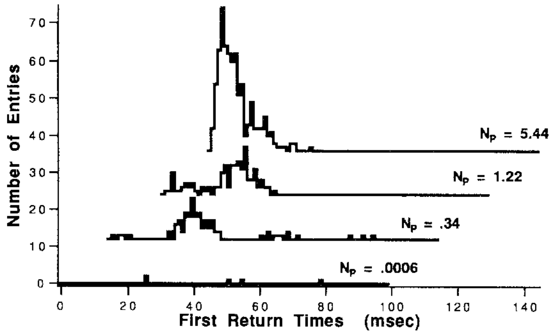

Fig. 6. First return times for some of the data in Fig. 4; the two central plots refer to data in the SR zone.

4. THEORY

The set of equations governing the LSA evolution[10] is

$$\dot{E} = -\frac{1}{2}\left(D + \frac{\bar{A}}{1 + a\,|E|^2} + 1\right) E + \zeta(t)$$
$$\dot{D} = -\gamma(D + A + D\,|E|^2) - c_1(S - D) \qquad (1)$$
$$\dot{S} = -\gamma_1(S - D)$$

where E represents the electric field amplitude, D the amplifier population difference, and S an auxiliary variable describing the population difference in other vibrational levels coupled to the amplifying one through collisions (the population difference in the absorber has been adiabatically eliminated), and $\zeta(t)$ is a Gaussian complex white noise due to spontaneous emission. A and $\bar{A}$ are control parameters of the strength of the amplifier and the absorber, respectively. We take that the correlation of $\zeta(t)$ is given by

$$\langle \zeta(t)\, \zeta^*(s) \rangle = 4T\delta(t - s)$$

where a factor 2 comes from assuming that the real and imaginary parts of $\zeta(t)$ are uncorrelated.

If an adiabatic elimination, justified by the different time scales in the real system, of the D and S variables is carried out, the electric field amplitude (for small values) evolves as

$$\frac{dE}{dt} = aE + b\,|E|^2\,E + c\,|E|^4\,E + \zeta(t) \qquad (2)$$

i.e., it moves in the potential

$$V(E) = -a\frac{|E|^2}{2} - b\frac{|E|^4}{4} - c\frac{|E|^6}{6} \tag{3}$$

This potential corresponds to a marginal stability case for the solution $E=0$, with two more solutions at $\pm E_S$.[14] The white noise is required to start the electric field evolution from the $E=0$ solution.

In the experiment, a modulation $F\cos(\omega t)$ and an external noise $\eta(t)$ were applied to the A parameter in the middle equation of equations (1), giving

$$A(t) = A + F\cos(\omega t) + \eta(t) \tag{4}$$

We will assume in the following that the noise $\eta(t)$ can be described as a white noise with zero average and autocorrelation

$$\langle \eta(t)\,\eta(s)\rangle = 2Q\delta(t-s) \tag{5}$$

although we should say that, on the time scale of the evolution of the variable D (the population difference) and S, the noise $\eta(t)$ is really perceived as heavily colored (i.e., with a Lorentzian spectral density with cutoff at frequencies much smaller than those involved in the evolution of D and S). In passing, we note that the LSA perturbed as per Eq. (4) with F a step function has been extensively studied also in ref. 15. The adiabatic elimination now leads to

$$\dot{E} = -\frac{E}{2}\left(1 + \frac{\bar{A}}{1+\alpha\,|E|^2} - \frac{A}{1+|E|^2} - \frac{\eta(t) + F\cos\omega t}{1+|E|^2}\right) + \zeta(t) \tag{6}$$

To understand the behavior of the system described by Eq. (6), particularly regarding the phenomenon of stochastic resonance, it is convenient to look at the corresponding Fokker–Planck equation without periodic modulation. Details of the calculation will be given elsewhere. Incidentally, a similar laser model, without periodic modulation and with only additive noise, has already been studied within a Fokker–Planck approach in ref. 16.

The final result is that the equilibrium distribution for the modulus of E becomes bistable in an appropriate range of parameters. The periodic modulation then induces switching between these two states, thus leading to SR in the system. The necessary ingredients are now the escape rates from these two states, which can be easily calculated. Finally, applying the linear response approach,[3] it is possible to calculate the response of

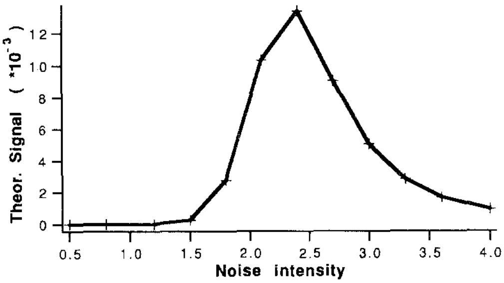

Fig. 7. Theoretical signal at the driving frequency (S) versus the intensity of the multiplicative noise Q. For the other parameters (see text) we have $A = 4.0$, $\bar{A} = 0.5$, $D = 1 \times 10^{-12}$, $\omega = 0.002$, $\alpha = 5$. Units are arbitrary.

the system as the various parameters are changed. We have plotted the expected response of the system (not the signal-to-noise ratio) as function of the amplitude of the multiplicative noise in Fig. 7. The overall shape is very similar to the analogous plot from the experiment.

5. CONCLUSIONS

In this work we investigated for the first time the onset of SR in a laser with saturable absorber operating in a bistable zone. We measured, as a function of the injected noise, the SNR and found that there is an optimum noise value for which the system jumps regularly from one stable state to the other.

We have also briefly accounted for the phenomenon from a theoretical point of view, using a well-established model for the LSA in its different behaviors.

We look forward to investigating experimentally the bistability in the presence of modulation and injected noise on the absorbing medium through a Stark cell and will make a quantitative comparison between theory and experiments.

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